So, H. J. et al. (Eds.) (2020). Proceedings of the 28th International Conference on Computers in Education.
Asia-Pacific Society for Computers in Education

# Design of a Game-based Intelligent Learning Environment to Remediate Fraction Addition/Subtraction Misconceptions

**Jarl Brent OBEDOZA[a*] & Raymund SISON[b*]**
[a,b]*College of Computer Studies, De La Salle University, Philippines*
[b]*College of Education, De La Salle University, Philippines*
*jarl_brent_obedoza@dlsu.edu.ph, raymund.sison@delasalle.ph

**Abstract:** Fraction addition and subtraction entail arithmetic procedures that are difficult for elementary students. Difficulties come from the inherent complexities of these procedures. A proposed method to deal with these complexities involves a game-based intelligent learning environment (GILE), the learning outcome mechanics and other game elements of which are designed based on the literature on fraction instruction and fraction misconceptions. Preliminary results seem to point toward the ability of GILE to improve learner performance and remediate misconceptions for gamers and those whose procedures are close to the mechanics.

**Keywords:** Game-based Intelligent Learning Environment, Educational Games for Fraction Addition/Subtraction, Remediating Fraction Addition/Subtraction Misconceptions

## 1. Introduction

Fraction arithmetic is difficult for elementary students. Difficulties they experience hinder learning of the proper procedures and concepts to perform fraction arithmetic (Lortie-Forgues et al., 2015). Because of these difficulties, misconceptions arise when students perform either fraction addition or subtraction, such as adding the numerators and denominators of 2 fractions separately (e.g. 1/2 + 2/3 = 3/5) (Aksoy & Yazlik, 2017; Siegler et al., 2011; Fazio & Siegler, 2011; Mohyuddin & Khalil, 2016). For this reason, teachers, researchers, and other concerned parties implement various techniques to improve students' learning of fraction arithmetic.

Some of the techniques involved are those used in the classroom environment. Some of these include the use of visuals, which make it easier for students to understand what is taught (Lamon, 2012; Nardi, 2014). An example of this is the use of bar models in Singapore math (Hoven & Garelick, 2007), where fractions are represented as concrete objects or pictures with associated numbers first before they are shown as abstract symbols. Physical manipulatives (Gabriel et al., 2012) take it a step further by allowing interaction. These help the students understand the mechanisms of fraction arithmetic in a concrete, tangible manner.

However, traditional classroom instruction has its limitations, such as the ratio of the teacher to the students (Schanzenbach, 2014). Teachers would have to cater to a group instead of the individual needs of students, causing inefficiencies when dealing with student misconceptions.

This issue can be addressed with the help of technology, such as intelligent tutoring systems (ITSs). ITSs solve this issue by providing students with individualized learning. They can guide students whenever they perform an error during the arithmetic process, allowing them to correct themselves (e.g. AnimalWatch, Beal, 2013). Systems also exist that make students commit errors (e.g. through trick questions) to diagnose problems for immediate correction (Layton, 2016). ITSs dealing with fraction arithmetic have been developed with good results (e.g., Beal, 2013; Riconscente, 2013). However, their explanations of fraction addition/subtraction procedures, if any, are through abstract symbols.

While ITSs can simulate tutors, the current generation of children learn differently than those of older generations (Prensky, 2001b). This indicates the need for a different approach, especially one that takes into consideration that which is familiar to the current generation, what they experience, and how they behave or think. Serious games, which are video games designed for productivity, provide individualized learning as well, and use engagement to motivate learning in an interactive learning environment, allowing the student to actively participate in the learning process, similar to how physical manipulatives are used.

A few serious games have been developed for fraction arithmetic that use visual models. Slice Fractions (Cyr et al., 2016) has been shown to enable students to learn subtraction of similar fractions, but not of fractions with different denominators, which necessitates the students' finding a common denominator. Discord (Espulgar et al., 2018) covers addition and subtraction of *dis*similar fractions, but its mechanics might be too complex for some learners.

For students to overcome the difficulties of fraction addition and subtraction, this research uses a game-based intelligent learning environment, a medium familiar to their generation, to implement an intuitive, comprehensible approach to alleviate difficulties by addressing misconceptions.

## 2. Game-based Intelligent Learning Environments for Fraction Addition/Subtraction

### *2.1 Game-based Intelligent Learning Environments*

An intelligent learning environment (ILE) is a computer program that adaptively provides a set of artifacts or tools that a learner can manipulate so that in the end, he or she will have learned a target concept or skill (Sison, 2001). To adapt to what the learner is currently doing or having difficulty with, an ILE uses a student model, also known as a learner model. A student model is a possibly partial and certainly approximate representation of the learner's knowledge, including misconceptions, as well as the learner's goals, preferences, and idiosyncrasies (Sison and Shimura, 1998; Woolf, 2010, p. 48). An intelligent tutoring system (ITS) is another kind of program for computer-assisted learning that makes use of a student model. It uses a student model to diagnose and remediate its learners. However, whereas an ITS performs the remediation explicitly, an ILE does this more subtly.

A game-based intelligent learning environment (GILE) is an ILE that is in the form of a videogame. It therefore falls under the category of a serious game or an educational game. However, it has the additional capability of understanding a user's misconceptions, if any, and can attempt to enable the user to recognize and correct these. Therefore, a GILE has three components: a game component, a learning modeling component, and a pedagogical component (Figure 1).

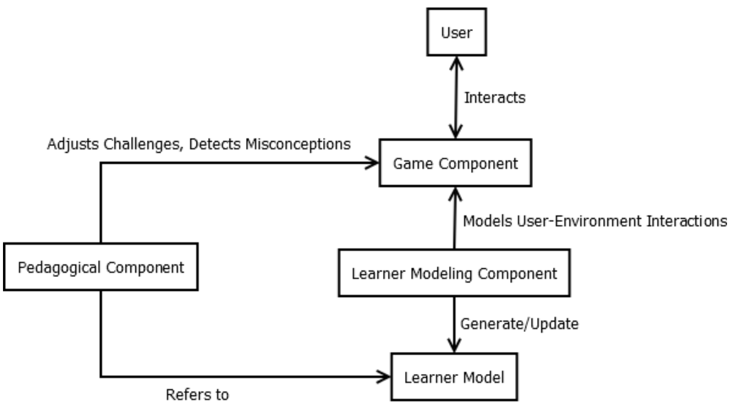


*Figure 1.* GILE Components and Interactions.

## 2.2 *Difficulties in Learning Fraction Addition and Subtraction*

Fraction addition and subtraction is difficult to learn, and Lortie-Forgues et al. (2015) have identified seven sources of difficulty found in fractions and decimal arithmetic, including the following.

1. Fraction Notation - Due to how a fraction is represented (in the form of a/b, b is not equal to 0), some students get confused and treat the numbers as independent numbers, as an arithmetic operation (e.g. a+b), or a single number (e.g. a/b is written as ab). This also increases cognitive load on the student when performing arithmetic operations.
2. Accessibility of Fraction Magnitudes – Understanding the magnitude of fractions is complex compared to that of whole numbers. Its factors (numerator and denominator) must be derived to get the representative magnitude, whereas whole number magnitudes can be understood as is. Knowledge of this has been statistically tested to be related to success in fraction arithmetic (Siegler et al., 2011).
3. Opaqueness of Standard Fraction Arithmetic Procedures – It is not apparent to some students why procedures during fraction arithmetic must be done. For example, why is there a need to make denominators equal before addition or subtraction can proceed? While there is an explanation, Lortie-Forgues et al. (2015) considers it advanced for the grade level where fractions are being taught (i.e. requiring algebra to explain).
4. Complex Relations between Rational and Whole Numbers Arithmetic Procedures - When adding or subtracting fractions, only the numerators are added or subtracted as if they were whole numbers but not the denominators, making the procedure confusing for students.
5. Sheer Number of Distinct Procedures - To perform addition or subtraction of fractions, one must ensure that the denominators are equal, making multiplication a necessary step, before the actual operation is done. This step requires mastery of fraction equivalence, which is a separate procedure than fraction addition or subtraction.

There are two other sources of difficulties, but they do are not directly related to fraction addition and subtraction.

These difficulties would explain why students commit errors. For example, they would respectively add the numerators and denominators of two fractions, an error associated with the misconception of numerator and denominator being two different independent natural numbers (Aksoy & Yazlik, 2017; Siegler et al., 2011; Fazio & Siegler, 2011; Mohyuddin & Khalil, 2016). A possible explanation for this is what Siegler et al. (2011) call the whole number bias, which is the knowledge of whole numbers interfering with the learning of fractions (Lamon, 2012). That is, students carry over knowledge from previous experiences and apply it to an inappropriate context (Layton, 2016), which points to the sources of difficulty identified (numbers 1 and 4 in the list above).

Siegler et al. (2011) report that adding numerators and denominators are more frequent in fraction addition/subtraction involving different denominators. This points to the difficulty students face when trying to make dissimilar fractions become similar. This is the procedure in the operation where fractions of different denominators are made to have similar denominators. For example, Idris & Narayanan (2011) and Abdul Ghani & Maat (2018) identified an unusual error where students would add numerators and choose one denominator as the denominator sum. Aksoy & Yazlik (2017) describes it as either not knowing how to perform denominator equalization or being unfamiliar with the operation. This points to the third and fifth difficulties in the list above.

Considering the situations stated above, a way to address the difficulty in equalizing the denominators of fractions is necessary. To do so, we must understand the operations to see why and how dissimilar fractions are made similar.

## 2.3 *Bar Models in Singapore Math*

Singapore math is based on James Bruner's theory that people learn in 3 steps: concrete, iconic, and abstract. They start by learning concepts through common objects, then associate these objects with abstract representation, before finally working with purely abstract representations. Bar models can be used in this manner by letting students work with visual objects to use procedures and operations on (e.g. addition can be taught by putting together bars of the same sizes). This way, students can focus on the underlying mechanisms of arithmetic operations without worrying about abstract notations. Abstract representations can then be gradually introduced, until the bar model can be removed. Slice Fractions (Cyr et al., 2016), a game for teaching fraction subtraction, has been shown to successfully use bar models through its game mechanics. However, students do not find common denominators of fractions by themselves. Discord (Espulgar et al., 2018) covers addition and subtraction of *dis*similar fractions, but its mechanics might be too complex for some learners.

## 3. System Design and Implementation

Endless Sky is the name of the educational mobile game presented in this paper. It is a game-based intelligent learning environment (GILE) for fraction addition and subtraction, developed in Unity 2019.1.5fl.

### *3.1 Outcome-based Game Design*

The design process of Endless Sky follows the iterative and incremental outcome-based game design methodology of Sison et al. (2018), which begins with, and is based on, intended learning outcomes (LOs). A learning outcome (LO) is a desired, tangible capability that a student can demonstrate after a learning experience (Spady, 1994). In an outcome-based educational approach, what the student learns at the end of the lesson determines what the curriculum and/or pedagogy is (Spady, 1994).

After the LOs have been identified, a suitable game genre and premise are then designed based on the LOs, after which special game mechanics, called LO mechanics are crafted.

To be effective, LO mechanics and the other formal and dramatic elements of a GILE must be designed with the help of established guidelines (e.g., Sweetser et al., 2017) as well as what the domain's literature suggests to be effective ways of teaching the domain, as well effective ways of addressing the major difficulties and misconceptions of learners in the domain, as discussed in Sections 2.2-2.4 above.

### *3.2 Learning Outcomes*

Endless Sky focuses on three learning outcomes regarding fraction addition and subtraction. These three are specified in the Department of Education's (DepEd's) Basic K to 12 Curriculum (Tabilang et al., 2015) and are listed in Table 1.

Table 1. *Fraction Addition/Subtraction Learning Outcomes*

| Learning Outcome | Associated DepEd Learner Competency Code |
|---|---|
| Correctly solve fraction addition and subtraction problems with similar denominators | M4NS-IIg-83 |
| Transform fractions with different denominators into fractions with similar denominators | M4NS-IIc-69.1 |
| Correctly solve fraction addition and subtraction problems that have different denominators | M4NS-IIg-83 |

### *3.3 Game Genre and Premise*

Endless Sky is an endless puzzle game. It is non-time-bound, so players can take their time solving the fraction addition and subtraction puzzles. The fraction addition and subtraction puzzles are randomly generated and become progressively challenging as the player progresses in the game. As it is an endless game, the player can continue playing until they trigger the game-over or feel like ending the session.

The game is set in a world with a vertically boundless sky that has, unfortunately, begun to collapse. Rifts in the sky have begun to form, causing sky fragments to fall. The player is tasked to prevent the sky from collapsing by sealing the rifts using the fallen sky fragments, ascending higher into the now distorted, endless sky.

The endless sky is actually made of a material that is made up of cells, which in turn can have any number of partitions. When a rift occurs, a fragment (i.e., a set of adjacent partitions) of a cell breaks off and falls to the earth. Each rift can therefore be viewed as a fraction and is represented visually using a bar model.

### *3.4 LO Mechanics and Other Game Elements*

To help students achieve the LOs shown in Table 1, Endless Sky uses four LO mechanics, shown in Table 2.

Table 2. *LOs and LO Mechanics*

| LO | LO Mechanic | Description |
|---|---|---|
| Correctly solve fraction addition and subtraction problems with similar denominators | Merge (for addition) | Combine two or more sky fragments with similar partitions. |
| | Cleave (for subtraction) | Use one sky fragment to detach pieces from another sky fragment with similar total pieces. |
| | Split (for subtraction) | Split a sky fragment into two smaller sky fragments. |
| Transform fractions with different denominators into fractions with similar denominators | Scale | Change the total pieces of a sky fragment based on a multiplier. |
| Correctly solve fraction addition and subtraction problems that have different denominators | Scale+Merge<br>Scale+Cleave<br>Scale+Split | |

The objects in the game that the user interacts with are the sky fragments that have fallen from the sky. Like rifts, fragments are fractions, and are also represented visually as bar models. To seal a rift, one can only use a fragment of the same size as the rift. So, a rift of 1/3 (of a sky cell) can only be sealed by a fragment with a value of 1/3 (of a sky cell). Attempting to seal a rift with the wrong-sized fragment will only create more rifts in the sky (as well as decrease the player's score). However, as Table 2 and Figure 2 show, it is possible to merge two sky fragments, split a sky fragment into two, cleave two sky fragments (detaching pieces of a sky fragment based on the number of pieces of another), or scale a sky fragment (not by changing its size but by changing the number of its partitions).


*Figure 2.* Tutorial for merging 2 or more sky fragments (top left), cleaving 2 sky fragments (top right), splitting a sky fragment into smaller ones (bottom left), and scaling sky fragments to adjust their total pieces (bottom right).

Figure 3 shows how the dissimilar fractions could be added. This corresponds to the third LO in Table 2, which involves two LO mechanics: scaling and either merging, cleaving, or splitting. In the figure, the rift 2/3 needs to be sealed, using the fragments 1/2 and 1/3. First, 1/2 is scaled to 2/4 (Figure 3-3), then 1/3 is scaled to 2/6 (Figure 3-4), then 2/4 is scaled to 3/6 (Figure 3-5), and the two are merged into 5/6 (Figure 3-6). Finally, 5/6 is split into 4/6 and 1/6 (Figure 3-7), in preparation for the scaling down of 4/6 to 2/3 and the eventual sealing of the rift 2/3. It should be noted that this is a more detailed step to show how the arithmetic process is adapted into game mechanics. Players can skip certain steps (such as 3-3) if they know, for example, that they should scale both fragments to have 6 total partitions. The game also allows, for example, the rift 2/3 with the fragment 4/6 to show fraction equivalence in case the player is knowledgeable about that concept.

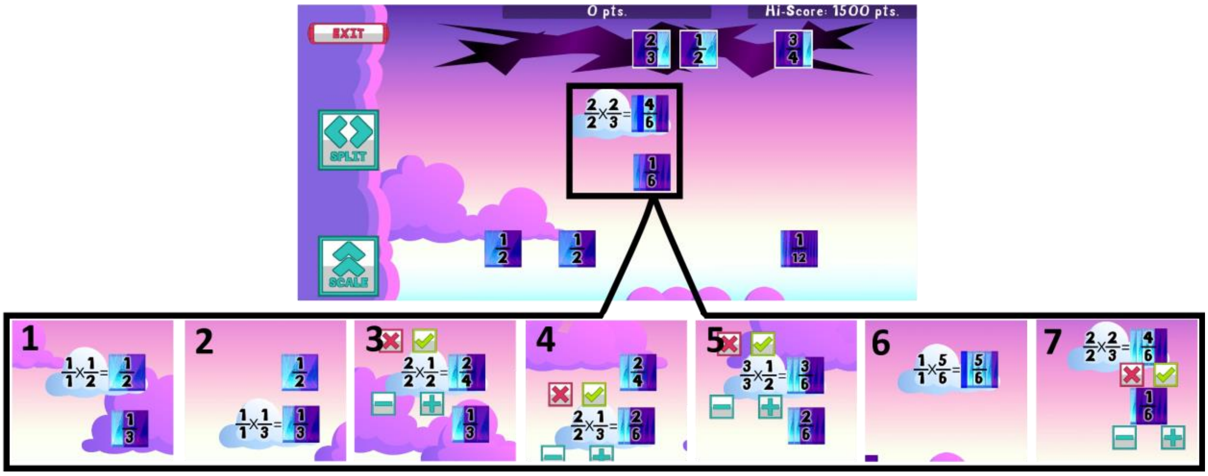


*Figure 3.* Scaling 2 dissimilar fractions represented as sky fragments in the game into similar fractions so they can be Merged into a new sky fragment that can be used to fill a rift the given blocks cannot fill.

As mentioned earlier, attempting to seal a rift with a fragment of the wrong size will only create more rifts. Moreover, a player can only incur one unsuccessful seal attempt per sky level. The second unsuccessful seal attempt will cause the sky level to collapse, bringing the sky, as well as the game, to an end. Of course, the student can always play again.

Sealing the rifts at a given sky level will enable the player to move up to the next sky level. The number of sky rifts that need to be sealed and the number of sky fragments that the user can work with are computed based on the player's performance, which is monitored by the learner modeling component, which will be discussed next.

Following the outcome-based design methodology, the difficulties that were enumerated in section 2.2 were also taken into consideration in the design of the other formal and dramatic elements of the game. Table 3 describes how these difficulties were addressed in Endless Sky.

Table 3. *Fraction Learning Difficulties and In-Game Elements that Address Them.*

| Difficulty | How the Game Addresses the Difficulty |
|---|---|
| Fraction Notation | The game transitions from showing pure visual representation of the fractions through the sky fragments, to visuals accompanied with the usual fraction notation, to only the fraction notation to allow the player to gradually understand fraction notation and see how they are visually represented. Through the LO mechanics (e.g. splitting, merging), the player is shown how the fraction notation and the visuals interact with respect to their actions (e.g. increasing or decreasing the numerator value, combining a group of sky fragments together). |
| Accessibility of Fraction Magnitudes | |
| Complex Relations between Rational and Whole Numbers Arithmetic Procedures | |
| Opaqueness of Standard Fraction Arithmetic Procedures | Visualization and hints using animation show players why groups of sky fragments cannot be merged/cleaved through emphasis on the differing number and sizes of the sky fragments' pieces and/or their denominator values. |
| Sheer Number of Distinct Procedures | The game lays down a simple rule that they cannot merge/cleave sky fragments with differing pieces and/or denominators and giving them freedom to work out the problem however they like. Being a video game, the LO mechanics streamline the arithmetic procedure, so the player need only worry about what steps to do to solve a problem, rather than worry about non-essentials. |

### 3.5 *Learner Modeling Component*

The learner model of Endless Sky is implemented using a Bayesian network (BN), which the learner modeling component manages. Figure 4 shows the BN's structure. Updates occur every time a puzzle is cleared, i.e., all rifts are sealed. The BN implementation used in Endless Sky reuses the probabilistic computation library used in (Madrigal et al., 2018) and Espulgar et al. (2018).

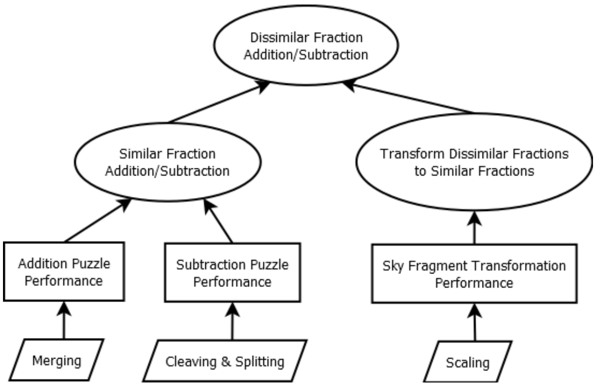


*Figure 4.* BN representing the Learner Model, with arrows indicating probability propagation direction.

In addition to the BN, Endless Sky also uses a table of misconceptions (Table 4). These misconceptions are based on Aksoy & Yazlik (2017), Siegler et al. (2011), Fazio & Siegler (2011), and Mohyuddin & Khalil (2016). When a player makes an unsuccessful attempt to merge/cleave fragments, Endless Sky tries to map the error into one of the two misconceptions in Table 4.

Table 4. *Misconceptions Table*

| Misconception | Error Variant | Example |
|---|---|---|
| Numerator and denominator are 2 independent natural numbers | Adding numerators and denominators independently | 2/3 + 1/3 = 3/6 |
| | Subtract numerators and denominators independently | 5/11 – 2/11 = 3/0 |
| Lack of knowledge/Unfamiliarity with process | Adding numerators and choosing one denominator as denominator sum | 14/15 + 2/30 = 16/30 |

Rifts spawning cause sky fragments to drop. Rifts are generated via random selection among a selection of integers. The range of the integers are determined by the observed performance of the user.

### 3.6 *Pedagogical Component*

The pedagogical component of Endless Sky uses the learner model to adjust the fraction values and whether the abstract fraction notation is shown together with the bar model or not. It decides the changes on the game component through threshold probabilities for each of the learner model's nodes. After the learner model is updated based on the recent performance of the player, the pedagogical component issues an inference request to the learner modeling component to retrieve the updated probabilities of the learner model. The nodes' rates (which represent the mastery of the LOs and other arithmetic skills) are used to decide how the puzzles are generated, making them easier or harder.

The pedagogical component also uses the learner model's knowledge of the player's misconceptions by generating "traps" that would "tempt" the player to commit an error that would expose a misconception shown in Table 4. For example, in Figure 4, the player must first solve a rift chosen by the game, which is the 2/3 rift. A set of sky fragments is generated to include some (notice the 1/1 and 1/2 sky fragments) which, when manipulated (using the LO mechanics) in a certain way (e.g. merging the 1/1 and 1/2 sky fragments without scaling them), would indicate the presence of a misconception. These traps are generated until the misconception rates are approximately 0%

(initialized at 100% each). Correctly solving puzzles laid with “traps” will gradually decrease the rates. Traps will not always be generated to prevent the player from becoming conscious of them.

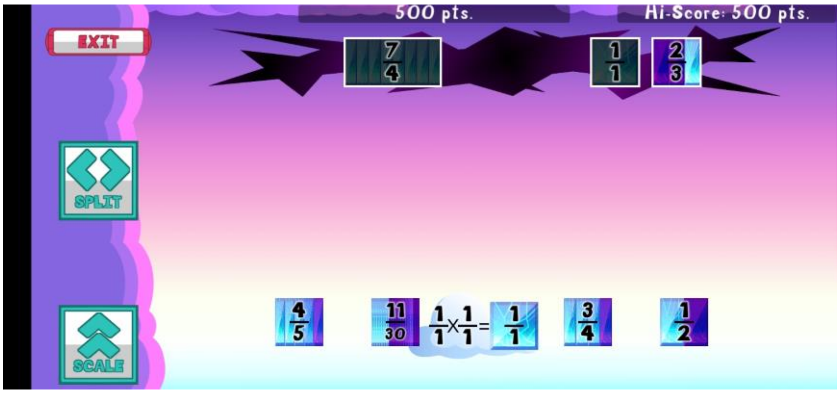


*Figure 5.* Testing a Misconception

Turning this component off causes the GILE to base its fraction value control and abstract fraction display on arbitrary game score milestones and alternating between trying a trap or not.

## 4. Preliminary Results

To evaluate the effectiveness of the GILE, especially the pedagogical component’s effectiveness in implementing its traps and using the learner model to control the puzzles, we planned to conduct a pretest-posttest quasi-experiment in an on-campus, in-person environment similar to what was done in (Sison, et al., 2018). However, the COVID-19 pandemic compelled us to conduct the pretest-posttests online. While waiting for a local school to conduct classes online, social media was used to gather Grade 5 or 6 participants. Participants were alternatingly assigned the active pedagogical component version and inactive version as purposively grouping participants who joined at different times was impossible. P1, P2, and P3 were the only ones that properly went through with the pretest, playtesting, and posttest.

When a local school (LHCS) began conducting a dry run of its online classes, we decided in coordination with a teacher to have the game playtested by its Grade 6 class of 19 students. Of the 19, 13 went with the pretest. The 13 were then grouped based on their pretest scores: 7 were assigned to playtest the game with the pedagogical component active (experimental group) and 6 were assigned to playtest the game with the pedagogical component turned off (control group). However, due to technical difficulties and compliance issues despite supervision of the teacher, only 4 of the 13 participants (1 from the experimental group, 3 from the control group) went through with the procedures properly.

In total, therefore, 7 participants were able to playtest the GILE and take the pretest and posttest. The results are shown in Table 6. P1, P2, and P3’s short quizzes had 8 items while P4, P5, P6, and P7 had 10 items. The difference in quiz items for the latter was at the consultation of the latter’s supervising online math teacher after she reviewed the short quizzes. For the playtesting of the GILE, participants were instructed to try beating a high score of 10,000 to get them working towards a goal when playing.

Table 6. *Preliminary Pretest-Posttest Results*

| Playtester | Pedagogical Component | Pretest Score | Posttest Score |
|---|---|---|---|
| P1 | Active | 50% | 100% |
| P2 | Inactive | 100% | 100% |
| P3 | Active | 37.5% | 37.5% |
| P4 | Active | 50% | 40% |
| P5 | Inactive | 60% | 80% |
| P6 | Inactive | 100% | 80% |
| P7 | Inactive | 70% | 70% |

Of those interviewed (P1, P4, P5, P6, P7), 4 found difficulty with the game. P4 had difficulty with the Scale mechanic. P6 took 7 minutes of his 17-19 minutes of play time to familiarize with the mechanics. P1 and P5 did not specify any mechanic but found the game "confusing at first" and "a bit hard", respectively. P7 found the game easy, scoring 57,300 points with 1 to 2 hours of play time.

P1's pre- and posttest scores suggest that he benefited from the GILE. An online chat with his older sister told us that he played the game for 80 minutes, and, being an avid video gamer, was motivated by the desire to beat the high score, which was 10,000, by scoring 12,000. Pretest data reveal that his misconception was "Numerator and denominator are 2 independent natural numbers" (recall Table 4). This was eradicated when, while playing the game:

> "he figured… that you cant merge or cleave blocks that arent of the same total number right? that's when he got it na dapat the denominators are the same before adding or subtracting"

P1 also enjoyed the game:

> "he beat the 10000 points. he got 12000 at 3:20pm, he started playing at 2pm. He said he likes the app so much hahaha"

P1 also used the in-game term, "scaling", in describing his fraction arithmetic procedure:

> "I imagined scaling the 4/5 then arriving at 8/10 then subtracted since it has the same denominator so the answer is 0." – when solving 8/10 - 4/5.

P3, P4, and P5 did not revise the numerator after changing the denominator, but only P5's posttest showed the removal of this erroneous pattern. P5 played the game for only 20-25 minutes with a high score of 4,000, giving up due to difficulty (note that she was playing the GILE with pedagogical component turned off). While she could not recall what she did differently in the posttest, she confirmed through interview that the GILE helped her understand fraction behavior during arithmetic and the Scale mechanic was no different to how she normally deals with dissimilar fractions. Her procedure:

> "1. multiply the denominator
> 2. do the cross multiply
> …
> example:
> 1/3 + 1/6
> …
> for the denominator times 6 times 3 = 18 so the common denominator is 18, 1 times 6 is 6 [and] 1 times 3 is 3, 6/18 + 3/18 = 9/18 or 1/2"

Her "cross multiply" step results in a number multiplying both numerator and denominator. This is close to how the Scale mechanic manipulates a multiplier which is used to multiply the sky fragment's base fraction value, i.e. numerator and denominator multiplied by the same number.

P4 had difficulty with the Scale mechanic. With only 10 minutes of play time, he would not have experienced the effects of the pedagogical component. Interview confirms that he does not revise the numerator after changing the denominators and does not see any error in that.

P2, P6, and P7 show no misconceptions in their short quiz results. Any mistakes found were not recurring and other items like the ones they got wrong were answered correctly. We could not get in touch with P2. P6 and P7 showed proper standard procedures for solving dissimilar fraction arithmetic.

These findings show the potential of the GILE to improve learner performance, particularly among gamers or those who can persevere past the game's difficulty, and those whose procedures are close to the LO mechanics.

## 5. Final Remarks

Fraction addition and subtraction entail arithmetic procedures that can be difficult for elementary students. In this paper, we have described a game-based intelligent learning environment (GILE), the learning outcome mechanics and other game elements of which were designed based on the literature on fraction instruction and fraction misconceptions. Preliminary results seem to point toward the ability of the GILE to improve performance and remediate misconceptions for learners who can thoroughly play the GILE and those whose procedures are close to the LO mechanics.

Our experience also reveals the difficulty of conducting quasi-experimental evaluations that are fully online. We plan to do more qualitative data collection and analysis in the near future.

## Acknowledgement

The researchers would like to thank DOST-SEI and ERDT for funding this research, the playtesters who volunteered to playtest the game, artists who contributed to the game's development, and friends who assisted in the pandemic-related difficulties that the researchers encountered.

## References

Abdul Ghani, S., & Maat, S. (2018). Misconception of fraction among middle grade year four pupils at primary school. Research on Education and Psychology, 111–125.

Aksoy, N. C., & Yazlik, D. O. (2017). Student errors in fractions and possible causes of these errors. Journal of Education and Training Studies, 5(11), 219– 233.

Beal, C. R. (2013). Animalwatch: An intelligent tutoring system for algebra readiness. In International handbook of metacognition and learning technologies (pp. 337–348). Springer.

Cyr, S., Riopel, M., & Charland, P. (2016, 4-6 July, 2016). Game design of a video game on fractions concept learning in elementary school students. In Edulearn16 proceedings (p. 4575-4582). IATED. Retrieved from http://dx.doi .org/10.21125/edulearn.2016.0210 doi: 10.21125/edulearn.2016.0210

Espulgar, C., Obedoza, J., Jazmine Veneth, S., & Samuel, R. (2018). Design of a game-based intelligent learning environment for elementary fractions. In Proceedings of the 26th international conference on computers in education.

Fazio, L., & Siegler, R. S. (2011). Teaching fractions. International Academy of Education.

Gabriel, F., Coch´e, F., Szucs, D., Carette, V., Rey, B., & Content, A. (2012). Developing children's understanding of fractions: an intervention study. Mind, brain, and Education, 6(3), 137–146.

Hoven, J., & Garelick, B. (2007). Singapore math: Simple or complex? Educational Leadership, 65(3), 28.

Idris, N., & Narayanan, L. M. (2011). Error patterns in addition and subtraction of fractions among form two students. Journal of Mathematics Education, 4(2), 35–54.

Lamon, S. J. (2012). Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers. Routledge.

Layton, R. D. (2016). Web-based diagnosis of misconceptions in rational numbers (Unpublished doctoral dissertation). University of Witwatersrand

Lortie-Forgues, H., Tian, J., & Siegler, R. S. (2015). Why is learning fraction and decimal arithmetic so difficult? Developmental Review, 38, 201–221.

Madrigal, L., Romblon, N., Si, M., Dimaunahan, R., & Sison, R. (2018). Design of a Game-Based Intelligent Learning Environment for Elementary Geometry. In *Proceedings of the 26th International Conference on Computers in Education*.

Mohyuddin, R. G., & Khalil, U. (2016). Misconceptions of students in learning mathematics at primary level. Bulletin of Education and Research, 38(1).

Nardi, E. (2014). Reflections on visualization in mathematics and in mathematics education. In Mathematics & mathematics education: Searching for common ground (pp. 193–220). Springer.

Prensky, M. (2001b). Digital natives, digital immigrants. On the horizon, 9(5).

Riconscente, M. M. (2013). Results from a controlled study of the ipad fractions game motion math. Games and Culture, 8(4), 186–214.

Schanzenbach, D. W. (2014). Does class size matter? Retrieved 2019-03-19, from https://nepc.colorado.edu/sites/default/files/pb - class size.pdf

Siegler, R. S., Thompson, C. A., & Schneider, M. (2011). An integrated theory of whole number and fractions development. Cognitive psychology, 62(4), 273– 296.

Sison, R. (2001). Framework for Intelligent Internet-Based Learning Support Systems. Proceedings of the Ninth International Conference on Computers in Education, (pp. 926-931).

Sison, R., Espulgar, C., Madrigal, L., Obedoza, J., Romblon, N., Sace, J., & Si, M. (2018). Toward an outcome-based methodology for developing game-based learning environments. In Proceedings of the 26th international conference on computers in education.

Sison, R., & Shimura, M. (1998). Student modeling and machine learning. International Journal of Artificial Intelligence in Education (IJAIED), 9, 128–158.

Spady, W. G. (1994). Outcome-based education: Critical issues and answers. ERIC.

Sweetser, P., Johnson, D., Wyeth, P., Anwar, A., Meng, Y., & Ozdowska, A. (2017). GameFlow in different game genres and platforms. *Computers in Entertainment (CIE)*, *15*(3), 1-24.

Tabilang, A. R., Arce, I. J. B., Pascua, R. V., Calayag, N. P., Dacuba, L. P., Borais, D. B., ... Daganta, J. A. D. (2015). Mathematics grade 4 teacher's guide. Retrieved 2019-07-28, from https://drive.google.com/ file/d/0B0d4eXfN5MXRYW9mSHhlekROTDA/view

Woolf, B. P. (2010). Building intelligent interactive tutors: Student-centered strategies for revolutionizing e-learning. Morgan Kaufmann.